Estimation and Forecasting Ability of Volatility Models in Two Emerging Stock Markets in Africa

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ABSTRACT

This project estimates four ARCH models and investigate their forecasting power for volatility of the rate of returns in two stock markets of emerging economies in Africa namely: Botswana Share Market Index for the period 20.11.2001 – 01.07.2014, and the Nairobi Securities Exchange 20 Share Index for the period 02.08.2006 – 01.07.2014. ARCH model, GARCH model, E-GARCH model and GJR-GARCH model were employed to analyze the rate of returns against two different distributions on error terms: normal distribution and student-t distribution. After comparing Akaike Information Criterion and the Schwarz Information Criterion, the best model was chosen to forecast the volatility of these emerging markets. It was found that the EGARCH model has the best forecast performance compared with other models for Nairobi Securities Exchange 20 Share Index. For the Botswana Share Market Index, the TARCH model was found to be the most appropriate model compared with other models .

Keywords: ARCH Models, forecasting power, stock markets, volatility of returns

1.INTRODUCTION:

Forecasting stock market returns volatility has great importance for both investors, traders as well as researchers, because predicting volatility might enable one to take risk-free decisions including portfolio selection and option pricing. Recent financial turbulence once again proved the importance of reasonable measurement of uncertainty in financial markets. This uncertainty is usually known as volatility which has crucial significance to financial decision makers as well as policy makers. Forecasting volatility has attracted the interest of many academicians; hence various models ranging from simplest models such as random walk to the more complex conditional heteroskedastic models of the GARCH family have been used to forecast volatility. GARCH was used to forecast volatility for the first time by Akgiray (1989). Over the years different variations of the GARCH model has been used to forecast volatility. These models include E-GARCH, GJR-GARCH, VS-GARCH, QGARCH .

Africa is in the process of becoming the new frontier for emerging market investors. The flow of investment into the entire continent is gathering pace to countries like Kenya, Botswana, Ghana and Zambia. Notable global firms such as the Russian-based investment bank Renaissance Capital (offices in Lagos and Nairobi), South Africa's Pamodzi Investment Holdings and London-based Blakeney Management have already invested billions of dollars in several emerging economies on the Black Continent.

The focus here is on Kenya and Botswana. One important attraction of investors to Africa is the increasingly qualified and sophisticated staff, with a new generation of internationally experienced African financiers who have big minds at the helm of most Africa-centered funds but high volatility in the stock market indices of these emerging markets presents uncertainty to the future.

Over the last few years, modelling and forecasting volatility of a financial time series have become a fertile area for research, this is simply because volatility is considered as an important concept for many economic and financial applications, like portfolio optimization, risk management and asset pricing. In simple words, volatility means "the conditional variance of the underlying asset return". A special feature of this volatility is that it is not directly observable, so that financial analysts are especially keen to obtain a precise estimate of this conditional variance process, and consequently, a number of models have been developed that are especially suited to estimate the conditional volatility of financial instruments, of which the most well-known and frequently applied model for this volatility are the conditional heteroscedastic models. The main objective of building these models is to make a good forecast of future volatility which will therefore, be helpful in obtaining a more efficient portfolio allocation, having a better risk management and more accurate derivative prices of a certain financial instrument.

Among these models, the Autoregressive Conditional Heteroskedasticity (ARCH) model proposed by Engle 1982 and its extension; Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model by Bollerslev 1986, and Taylor 1986 were found to be the first models introduced into the literature and have become very popular in that they enable the analysts to estimate the variance of a series at a particular point in time Enders 2004. Since then, there have been a great number of empirical applications of modelling the conditional variance of a financial time series, Bollerslev et al. 1992, Engle and Patton 2001 , Shin 2005, Alberg et al. 2008, Shamiri and Isa 2009 and Kalu 2010. These types of models were designed to explicitly model and forecast the time-varying conditional second order moment (variance) of a series by using past unpredictable changes in the returns of that series, and have been applied successfully in economics and finance, but more predominantly in financial market research.

A lot of empirical studies have been done on modelling and forecasting stock market volatility by application of ARCH – GARCH specifications and their large extensions. Most of these studies focus on developed markets, and to the best of my knowledge, not much has been done on emerging markets especially in Africa. Wei Jiang use the GARCH model to analyze and predict the conditional variance and appropriate model for five global stock markets of developed nations, Lars Karlsson fit the GARCH model into financial time series using several distributions; Mohammed Aminul islam estimates volatility of three Asian markets with symmetric GARCH models to mention a few. In this work however, focus will be on two emerging African markets namely: Botswana Share Market Index, and the Nairobi Securities Exchange 20 Share Index. The aim is to analyze and estimate volatility of the rate of returns in these two emerging markets in Africa using several volatility models of the GARCH family

BACKGROUND OF THE STUDY

Stock Market is the term for overall market in which shares are issued and traded on exchanges or in over-the-counter markets. Also known as equity market, it is one of the most vital areas of a market economy because it provides companies with access to capital and allows investors to own companies and participate in economic growth.

The stock market serves the economy of the nation in many ways, for instance, it serves as a source of fresh external capital for companies and government enhancement, it offers access to a variety of financial institutions that enable economic agents to pool, price and exchange stocks. The stock market also provides securities for saving and investment and it also helps the government in the implementation of policies in relation to the redistribution of ownership of industries such as indigenization, privatization and commercialization schemes. It implies that the stock market refers to the network of the institution and mechanism for buying and selling of stock.

The instability in the daily returns securities can be considered as one of the consequences of the stochastic nature of the financial markets. In the face of usual less reliable price movements, investors invest their funds in the financial markets particularly in the stocks or stock indices with the expectation of being compensated by risk-premium. The variation provided by the stocks due to changes in the daily returns is generally termed as volatility which is measured by the standard deviation or the variance The usual fluctuations of the rally of the stock prices may not be bad but it ends up bad if the price swing are unusually very sharp or rapid over short time periods as it makes financial planning difficult. Higher fluctuations in the prices obviously increase the uncertainty about the future returns and hence increase the risk. If the market in the returns performance is unstable, investors cannot reliably predict the future which may result in further uncertainty about future price movements.

SCOPE OF THE STUDY

This project research focuses on the analysis of two emerging African markets namely: Botswana Share Market Index, and the Nairobi Securities Exchange. The study covers the behavior of these markets, predicts their conditional variance and the reliability and forecasting performance of the models used. It is noteworthy that the study only made use of existing theories and empirical findings.

LITERATURE REVIEW

EMERGING ECONOMIES IN AFRICA, STOCK MARKET AND VOLATILITY MODEL

An emerging market is a country that has some features of a developed market, but does not meet standards to be a developed market. This includes countries that may be developed markets in the future or were in the past. The economies of China and India are considered to be the largest. Emerging market hedge fund capital reached a record new level in the first quarter of 2011 of $121 billion. The four largest emerging and developing economies by either nominal or PPP-adjusted GDP are the BRIC countries (Brazil, Russia, India and China). The next four largest markets are Mexico, Indonesia, Turkey, and Saudi Arabia. Goldstein, Andrea et al (2006)

Currently, there are 29 formal stock markets in Africa, and with further proposals to open new ones in a number of African countries (Moin, 2007;Data bank Group, 2008; ASEA, 2012). The apparent substantial increase in stock markets in Africa can be attributed to the extensive financial sector reforms undertaken by a number of African countries (Kenny and Moss, 1998;Ramb accussing, 2010). It has been suggested that stock markets promote economic growth. For example, Schumpeter (1911), McKinnon (1973), Shaw (1973), Levine and Zervos (1996), and Levine (1997), amongst others, have argued that well developed capital markets can promote economic growth through their ability to attract international investments, mobilize domestic savings, provide liquidity, and hence, facilitate efficient allocation of scarce economic resources. However, despite the rapid development in the establishment of stock markets in Africa, with the exception of South Africa, stock markets in Africa not only remain comparatively different from their developed counterparts, but also, pale into insignificance in comparison to other emerging markets (Alagidede, 2009;2010; Ntim et al, 2011).

First, they are small in size (Kenny and Moss, 1998; Ntim et al, 2011). The total value of African stocks outside of South Africa was only 0.94% of world stock market capitalization, and 2.14% of all emerging markets stocks at the end of 2011 (World Federation of Exchanges (WFEs, 2012)). Similarly, African markets excluding South Africa accounted for only 3.46% of the total global equity listings in contrast to 12.29% by India for instance alone (WFEs, 2012).
More so, the stock markets are also small compared with the size of their own economies (Kenny and Moss, 1998; Ntim et al, 2011). For example, market capitalization in Mozambique is only 4.7% of nominal GDP, whilst Nigeria, Uganda and Tunisia's capitalizations are between 31-63% (WFEs, 2012). These figures are not only much less than developed markets, such as UK (145.6%), and US (122.8%), but also other emerging markets, such as Malaysia (183.7%), India (172.5%) and Brazil (110.8%) (WFEs, 2012).

Furthermore, their small size makes them vulnerable to speculation and manipulation (Magnusson and Wydick, 2002; Ntim et al, 2011), by insiders at the expense of other investors. More critically, they remain extremely illiquid, and thinly traded, severely affecting their informational efficiencies (Mlambo and Biekpe, 2005; Ntim et al, 2007, 2011). However, their ability to effectively perform the above listed roles depends heavily on their level of allocative, operational, and in particular, informational efficiency (Kenny and Moss 1998; Smith et al, 2002).
Africa's strategic partnership with China, India, and other emerging economies since the late 1990s has had a noticeable impact on the growth of trade and overall economic performance on the continent.

EMPIRICAL REVIEW

Several empirical works have been done since the seminar paper of Engel(1982) on volatility modelling, especially in finance, even though a number of theoretical issue are still unresolved (see Franses and McAleer, 2002).However, Anders (2006) believes that previous research on the effects of error distribution assumptions on the variance forecasting performance of GARCH family models is scarce. Some of the work on volatility modelling estimate a particular GARCH model with one or two error distributions, while some applied a particular error distribution to few ARCH family models to either establish the best forecasting model for conditional variance, the best fitted volatility model or confirm the ability of the models to capture stylized fact inherent in high frequency financial time series.

To the knowledge of this study, research on the contribution of error assumptions on volatility modeling in Africa is extremely minimal. Available literatures tend to capture the asymmetric properties of financial data without recourse to error distributions. Jayasuriya (2002) examines the effect of stock market liberalization on stock return volatility using Nigeria and fourteen other emerging market data.from December 1984 to March 2000 to estimate asymmetric GARCH model. The study inferred that positive (negative) changes in prices have been followed by negative (positive) changes.

The Nigerian session of the result tilted more to business cycle of behavior of return series than volatility clustering. Ogum *et al.* (2005) apply the Nigeria and Kenya stock data on EGARCH model to capture the emerging market volatility. The result of the study differed from Jayasuriya (2002). Though volatility persistence is evidenced in both market; volatility responds more to negative shocks in the Nigeria market and the reverse is the case for Kenya market.

Dallah and Ade (2010) examine the volatility of daily stock returns of Nigerian insurance stocks using twenty six insurance companies' daily data from December 15, 2000 to June 9 of 2008 as training data set and from June 10 2008 to September 9 2008 as out-of-sample dataset. The result of ARCH (1), GARCH (1, 1) TARCH (1, 1) and EGARCH (1, 1) shows that EGARCH is more suitable in modelling stock price returns as it outperforms the other models in model evaluation and out-of-sample forecast. Okpara and Nwezeaku (2009) randomly selected forty one companies from the Stock Exchange to examine the effect of the idiosyncratic risk and beta risk on returns using data from 1996 to 2005. By applying EGARCH (1, 3) model, the result shows less volatility persistence and establishes the existence of leverage effect in the stock market, implying that bad news drives volatility more than good news. Ngozi Atoi(2008)

GARCH, Generalized Autoregressive Conditional Heteroskedastic, models have become important in the analysis of time series data, particularly in financial applications when the goal is to analyze and forecast volatility. For this purpose, the family of GARCH functions offers functions for simulating, estimating and forecasting various univariate GARCH-type time series models in the conditional variance and an ARMA specification in the conditional mean, Diethelm Wuertz and Yohan Chalabi,2015. The number of GARCH models is immense, but the most influential models were the first, the standard ARCH model introduced by Engle [1982] and the GARCH model introduced by Bollerslev [1986], others includes the more general class of asymmetric power ARCH models, named APARCH, introduced by Ding, Granger and Engle [1993]. The APARCH models include as special cases the TS-GARCH model of Taylor [1986] and Schwert [1989], theGJR-GARCH model of Glosten, Jaganathan, and Runkle [1993], the T-ARCH model of Zakoian[1993], the N-ARCH model of Higgins and Bera [1992], and the Log-ARCH model of Geweke[1986] and Pentula [1986].

Forecasting conditional variance with asymmetric GARCH models has been comprehensively studied by Pagan and Schwert (1990), Brailsford and Faff (1996) and Loudon et al. (2000). A comparison of normal density with non-normal ones was made by Baillie and Bollerslev (1989), McMillan, et al. (2000), Lambert and Laurent (2001), Jun Yu (2002) and Siourounis (2002).

. Emerson et al. (1996) investigate Bulgarian stock market and Scheicher (1999) studies Polish stock returns, Shields (1997) modeling returns for the Warsaw and Budapest stock exchanges returns. Also Scheicher (2001) analyses the movements of the short rates of emerging markets in Central and Eastern Europe. Scheicher (2001) called Hungary, Poland and Czech markets as a principal emerging stock markets in Europe. The author estimate a VEC model and modeling its volatility with a Multivariate GARCH (M-GARCH) model. The findings shows that countries which are investigated have limited interaction and their volatility have a regional character.

Vošvrda and Žikeš (2004) study the behavior of volatility and the distributional properties of the Czech, Hungarian and Polish stock markets data for the period 1996- 2002 period using weekly data. They use PX-50 index for Czech Republic and find statistically significant results for GARCH (1,1) model and conclude that the volatility of the returns on PX-50 is very persistent.

METHODOLOGY

ARCH model

Engle (1982) proposed the ARCH model (Auto-regressive Conditional Heteroskedastic Model).

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

Baillie and Bollerslev (1989) explained the variation on error terms has been changed from the constant to be a random sequence. Teräsvirta (2006) pointed out, ε_t has a conditional mean and variance based on the information set I_{t-1}.

$$E(\varepsilon_t | I_{t-1}) = 0$$

$$\sigma_t^2 = E(\varepsilon_t^2 | I_{t-1})$$

Here,

$$\varepsilon_t = z_t \sigma_t$$

$$z_t \sim N(0,1)$$

So $\{\varepsilon_t\}$ is a normal distribution which mean equals to zero and variance equals to σ_t^2,

$$\varepsilon_t \sim N(0, \sigma_t^2),$$

Assume that, $\alpha_0 > 0$ and $\alpha_i \geq 0$ $i = 1, \ldots, q$, $\alpha_1 + \cdots + \alpha_q < 1$ for ensuring that $\{\sigma_t^2\}$ as weak stationary.

Generalized-ARCH model (GARCH)

Bollerslev (1986) and Taylor (1986) proposed the so-called generalized ARCH (GARCH) model for substituting the ARCH model.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

Alexander and Lazar (2006) assume $\alpha_0 > 0$ and $\alpha_i \geq 0,\ i = 1, \dots, q;\ \beta_j \geq 0,\ j = 1, \dots, p;\ \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 < 1$ for ensuring $\{\sigma_t^2\}$ as weak stationary. Enocksson and Skoog(2012) pointed out some limitations on GARCH model. The most important one is GARCH model cannot capture the asymmetric performance.

Later, for improving this problem, Nelson (1991) proposed the EGARCH model and Glosten, Jagannathan and Runkel (1993) proposed GJR-GARCH model.

Exponential GARCH (EGARCH) model

Nelson (1991) proposed the exponential GARCH (EGARCH) model.

$$\log \sigma_t^2 = c + \sum_{i=1}^{p} g(Z_{t-i}) + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^2 ,$$

Where,

$$g(Z_{t-i}) = \gamma_i Z_{t-i} + \alpha_i(|Z_{t-i}| - E(|Z_{t-i}|))$$

Define, $Z_{t-i} = \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$ and the nature logarithm of the conditional variance equals to:

$$\log(\sigma_t^2) = c + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i(|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}| - E(|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}|)) + \sum_{i=1}^{p} \gamma_i \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

Alexander (2004) presented represents the symmetric effect, β measures the lagged conditional variance and γ reflects the asymmetric performance.

$$E(|Z_{t-i}|) = \begin{cases} \sqrt{\frac{\pi}{2}}, & when\ Z_{t-i}\ is\ normal\ distribution \\ \frac{\sqrt{v}\Gamma[0.5(v-1)]}{\sqrt{\pi}\Gamma[0.5v]}, & when\ Z_{t-i}\ is\ student-t\ distribution \end{cases}$$

Wang, Fawson, Barrett and Mcdonald (2001) demonstrate $E(|Z_{t-i}|)$ is constant for all i

when Z_t is normal distribution or is $\frac{\sqrt{v}\Gamma[0.5(v-1)]}{\sqrt{\pi}\Gamma[0.5v]}$ depended on different when Z_t is student-t distribution.

GJR-GARCH model

Glosten, Jagannathan and Runkle (1993) proposed GJR-GARCH model, another asymmetric model also called the TARCH model. Define the sequence $\{\varepsilon_t\}$ equals to $z_t\sigma_t$ and $\{\varepsilon_t\}$ is a normal distribution.

$$\varepsilon_t \sim N(0, \sigma_t^2),$$

So the GJR-GARCH model is written by

$$\sigma_t^2 = c + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{k=1}^{r} \gamma_k \varepsilon_{t-k}^2 \, \mathrm{I}_{t-k}(\varepsilon_{t-k} < 0)$$

In GJR-GARCH model, the sign of the indicator term captures the asymmetry and Patrick, Stewart and Chris (2006) describes it in details in their article.

$$\mathrm{I}_t = \begin{cases} 1, & if\, \varepsilon_t < 0 \\ 0, & otherwise \end{cases}$$

Where I_t is an indicator function, when the residual (ε_t) is smaller zero, the indicator term (I_t) equals to one or equals to zero when the residual is not smaller than zero.

Distribution of the error term

This research mainly introduces two distributions. One is normal distribution, the other one is student-t distribution.

Normal distribution

The probability density function of Z_t is given as follows,

$$f(\mathrm{Z}_t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{\mathrm{Z}_{t-\mu}}{\sigma}\right)^2\right\}$$

Where μ is mean and σ is standard deviation.

Student –t distribution

The probability density function of Z_t is given as follows,

$$f(\mathrm{Z}_t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{(v-2)\pi}} \left(1 + \frac{\mathrm{Z}_t^2}{v-2}\right)^{-\frac{1}{2}(v+1)}$$

Where v is the number of degree of freedom, $2 < v \leq \infty$, and Γ is gamma function. When $v \to \infty$, the student-t distribution nearly to the normal distribution. The lower the v, the fatter the tails.

4.1 DATA PRESENTATION

The data used for this project work was culled from yahoo finance historical databank. The data comprises the Botswana Share Market Index historical data from 20 November 2001 to 31 June 2014 and the Nairobi Securities Exchange Index (NSE20) historical data from 2 August 2006 to 31 June 2014. Presented here is a graphical presentation of data and application of ARCH, GARCH, E-GARCH and TARCH models with two different error distributions; normal distribution and the student's t distribution.

4.2 DATA ANALYSIS

Fig. 1 : Graphs of Daily Returns Data for BSMI and NSE20 Stocks

The above graphs show some pattern of trend which suggests that the series are non-stationary. It can be seen that the graphs above show volatility in the return series over the sample period.

Fig. 2: Graphs of the Differenced Series showing no pattern of Trend

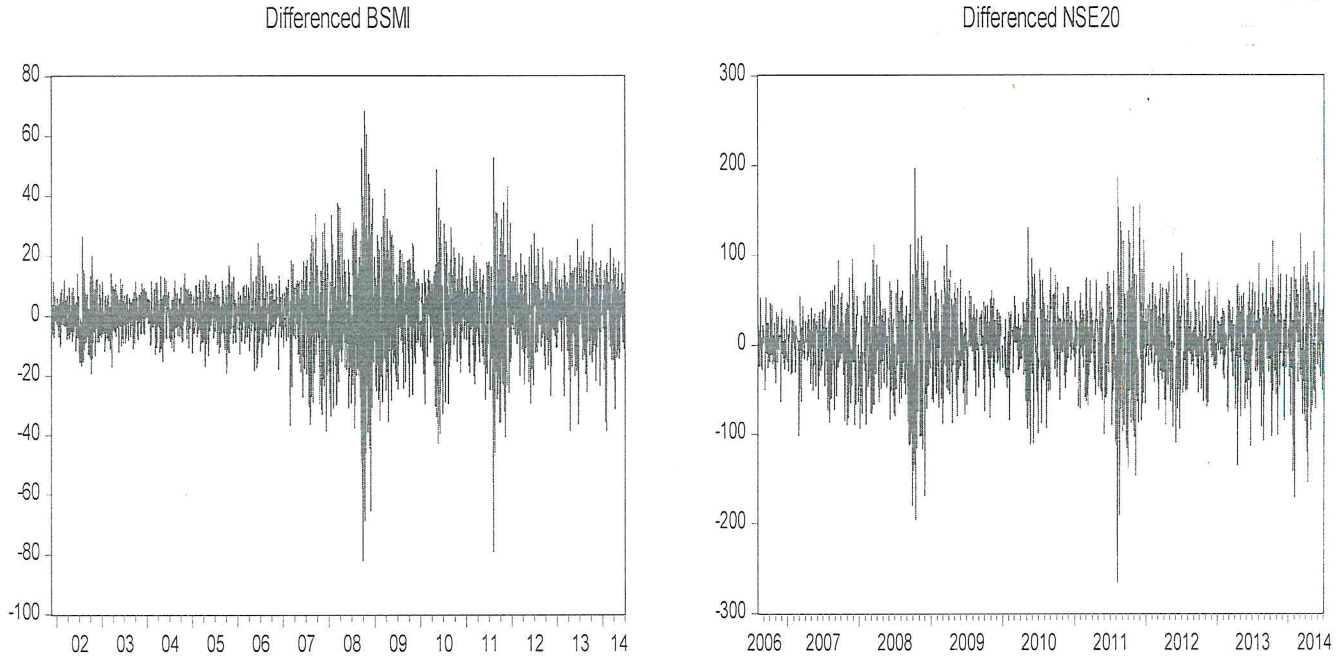


Table 1: Descriptive statistics of each of the average daily Indices.

	Table2: SUMMARY STATISTICS OF THE NIGERIAN STOCK RETURNS									
	Sample size	**Mean**	**Median**	**Min.**	**Max.**	**Std. Dev.**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**Probability**
NSE20	2064	1.44999	3.765	-265.81	197.1	42.48054	-0.4532	5.647521	673.4605	0.0000
BSMI	3290	0.33744	0.975	-82.15	68.42	11.3507	-0.5885	8.5513	4414.362	0.0000

From this table, the skewness is -0.4532 and -0.5885. They are not zero which means all of the rates of returns are not symmetric. The kurtosis for different rate of returns is 5.6475 and 8.5513. They are larger than three, which means the stock returns have the fat-tail characteristic. Furthermore, the Jarque Bera Test tells us the high values represent the non-normality of the rate of returns. From the table, JB test have enough large values (4414.362, 673.4605), so the rate of returns are not normally distributed.

4.2.1 Result

In this part, GARCH models are used to estimate the different rate of returns then compare the outcomes and choose the appropriate model to estimate and forecast the volatility of each market. The criterion AIC and SIC is used to choose the appropriate model. The lower the AIC and SIC the better fitted is the model.

4.2.2 Application to NSE20 daily return

The residual of the regression model was estimated using the method of least squares and the graph of the residuals is presented below.

Table 2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3167.106	20.09389	157.6154	0.0000
R-squared	0.000000	Mean dependent var		3167.106
Adjusted R-squared	0.000000	S.D. dependent var		897.0522
S.E. of regression	897.0522	Akaike info criterion		16.43661
Sum squared resid	1.60E+09	Schwarz criterion		16.43942
Log likelihood	-16378.08	Hannan-Quinn criter.		16.43764
Durbin-Watson stat	0.002360			

Residual of the regression model

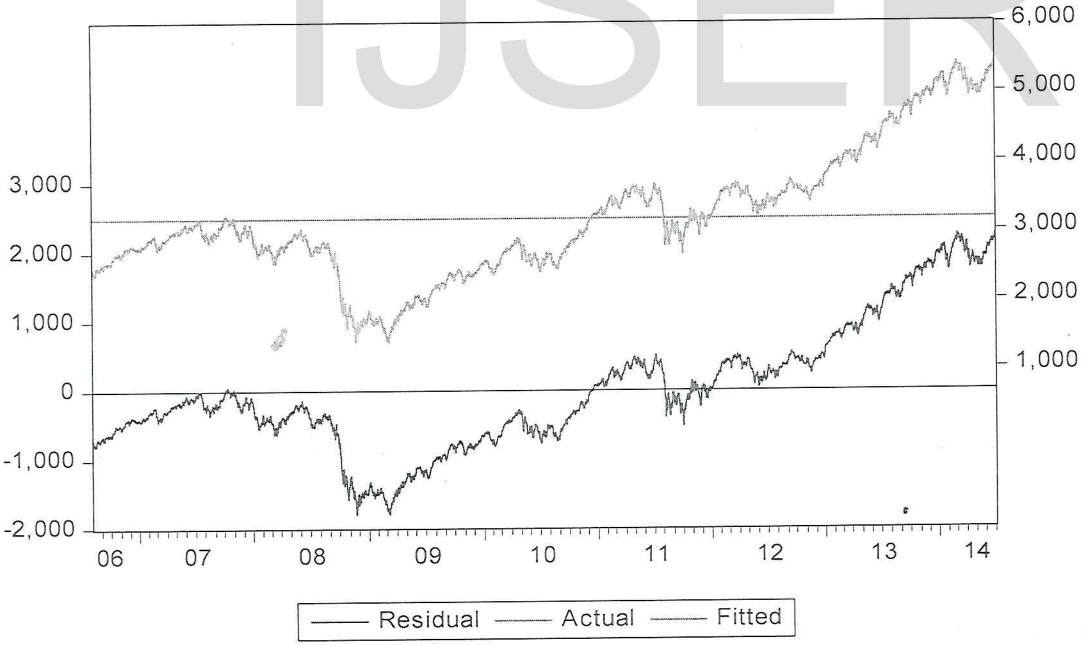


figure 5

From the table, periods of low volatility tends to be followed by periods of low volatility for a prolonged period and vice versa. When this happens for the residual, then there is every justification to run ARCH family model. We can however further confirm this by employing the ARCH test as stated below.

ARCH test result is as follows:

Table 3

Heteroskedasticity Test: ARCH

F-statistic	64.56305	Prob. F(1,2061)	0.0000
Obs*R-squared	62.66273	Prob. Chi-Square(1)	0.0000

Based on the assumption of 5% significance level, p = 0.0000 is less than 0.05 meaning reject the null hypothesis and accept the alternative hypothesis stated below

H_0: There is no ARCH effect vs H_1: There is ARCH effect

As a result, ARCH family models can be used to estimate the volatility of stock market indices.

			Normal Distribution				Student's t Distribution			
Models	Equations	Model Parameter	Coefficients	P-Value	SIC	AIC	Coefficients	P-Value	SIC	AIC
ARCH(5)	Mean	Intercept	2.26804	0.015	10.316	10.3078	3.64744	0	10.241	10.23
	Variance	Intercept	1506.813	0			1544.59	0		
		Resid(-1)^2	0.168315	0			0.19118	0		
GARCH (1, 1)	Mean	Intercept	2.737893	7E-04	10.133	10.1221	3.63774	0	10.119	10.10
	Variance	Intercept	29.43232	0			20.6485	0.007		
		Resid(-1)^2	0.076389	0			0.07678	0		
		GARCH(-1)	0.906849	0			0.91372	0		
TGARCH (1, 1)	Mean	Intercept	1.611569	0.039	10.1089	10.0953	2.54242	7E-04	10.099	10.08
	Variance	Intercept	41.24901	0			35.3526	0		
		Resid(-1)^2	-0.02148	0.046			-0.02378	0.076		
		Resid(-1)^2 *(RESID(-1) <0)	0.154323	0			0.16044	0		
		GARCH(-1)	0.914142	0			0.91578	0		

EGARCH (1, 1)	Mean	Intercept	1.233608	0.111	10.1026	10.0889	2.28219	0.002	10.094	10.07
	Variance	Intercept	0.174385	0			0.12967	6E-04		
		Resid(-1)^2	0.10693	0			0.10799	0		
		GARCH(-1)	-0.13815	0			-0.14165	0		
		LOG(GARCH(-1)	0.964615	0			0.97015	0		

From the table above, the EGARCH (1, 1) model has the minimum AIC and SIC , and therefore is the most appropriate model for Nairobi Securities Exchange Index under the student-t distribution and normal distribution.

Application in BSMI daily return

Find the residual of the regression model

Table 9

Dependent Variable: BSMI

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	957.1031	4.791196	199.7629	0.0000

Residual of the regression model

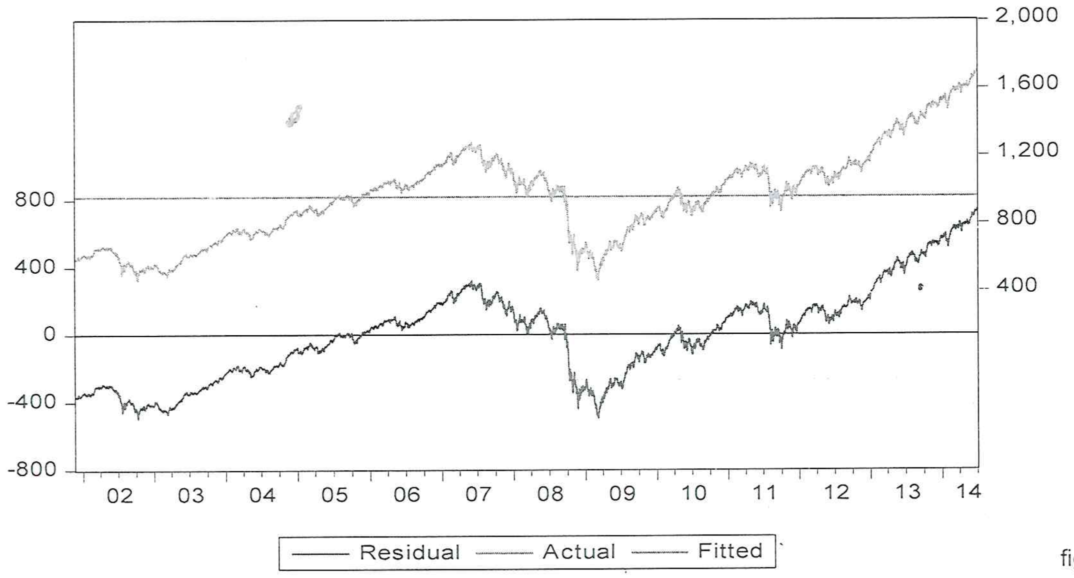


figure 6

From the table, periods of low volatility tends to be followed by periods of low volatility for a prolonged period and vice versa. When this happens for the residual, then there is every justification to run ARCH family model. We can however further confirm this by employing the ARCH test as stated below.

ARCH test result is as follows:

Table 10
Heteroskedasticity Test: ARCH

F-statistic	817288.7	Prob. F(1,3059)	0.0000
Obs*R-squared	3049.586	Prob. Chi-Square(1)	0.0000

Based on the assumption of 5% significance level, from the table, $p = 0.0000$ is less than 0.05 meaning reject the null hypothesis and accept the alternative hypothesis stated below

H_0: There is no ARCH effect vs H_1: There is ARCH effect

Now using ARCH family models to estimate the volatility of BSMI stock returns, we have the following;

Table 4: Estimation Results of First Order GARCH Family Models

			Normal Distribution				Student's t Distribution			
Models	Equations	Model Parameter	Coefficients	P-Value	SIC	AIC	Coefficients	P-Value	SIC	AIC
ARCH(1)	Mean	Intercept	0.555021	0.002	7.63278	7.62722	0.95197	0	7.4504	7.443
	Variance	Intercept	98.3827	0			101.415	0		
		ARCH (1)	0.274183	0			0.4195	0		
GARCH (1, 1)	Mean	Intercept	0.601622	0	7.27231	7.26489	0.79904	0	7.2532	7.243
	Variance	Intercept	0.752306	0			0.71286	0		
		ARCH (1)	0.07771	0			0.07854	0		
		GARCH(-1)	0.917274	0			0.9172	0		
TGARCH (1, 1)	Mean	Intercept	0.399339	0.004	7.26034	7.25107	0.62165	0	7.2407	7.229
	Variance	Intercept	0.987681	0			0.80642	0		
		ARCH (1)	0.014602	0.117			0.00501	0.646		
		Resid(-1)^2 *(RESID(-1) <0)	0.091898	0			0.10258	0		
		GARCH(-1)	0.927631	0			0.93236	0		

EGARCH (1, 1)	Mean	Intercept	0.340035	0.015	7.26514	7.25588	0.59003	0	7.2443	7.233
	Variance	Intercept	-0.04432	3E-04			-0.0509	8E-04		
		ARCH (1)	0.139327	0			0.13132	0		
		GARCH(-1)	-0.07308	0			-0.08418	0		
		LOG(GARCH(-1)	0.985918	0			0.98826	0		

From the table above, the TGARCH (1, 1) model has the lowest values of AIC and SIC respectively in the BSMI daily returns and therefore is the most appropriate model for the Botswana Share Market Index under the student-t distributions.

Diagnostic Checking

H_0: There is no ARCH effect vs H_1: There is ARCH effect

It is noteworthy to know that it is not serially correlated since the p-value of the correlogram squared residual is above 5% significance level which is desirable. Here, using ARCH LM test, we accept the null hypothesis since the p-value is greater than the 5% significance level which is also desirable.

Table 15 Heteroskedasticity Test ARCH

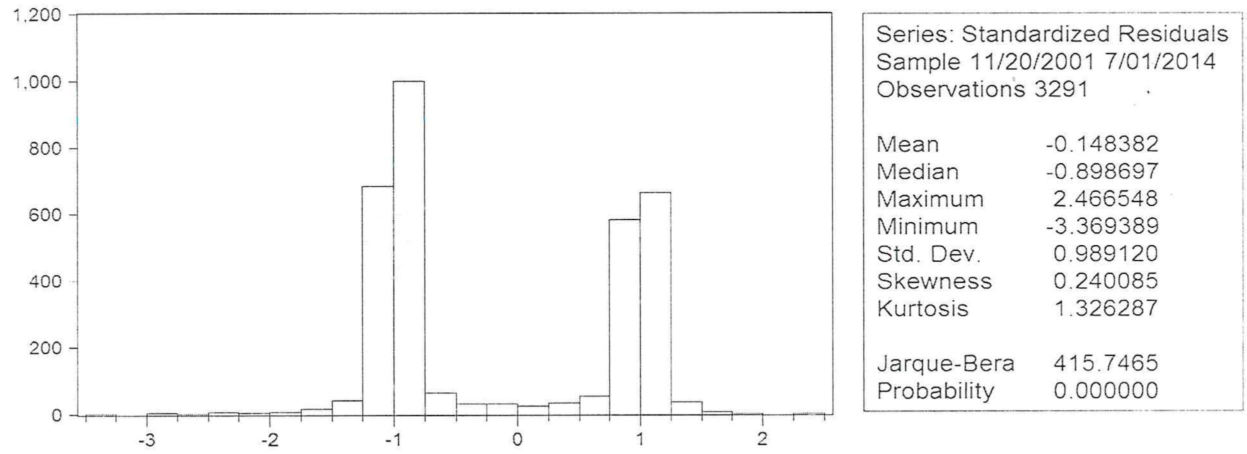
F-statistic	0.669042	Prob. F(1,3288)	0.4134
Obs*R-squared	0.669313	Prob. Chi-Square(1)	0.4133

And secondly, check whether the residual are normally distributed or not

H_0: Residuals are normally distributed vs H_1: Residuals are not normally distributed

The p-value below is 0.0000 thereby rejecting the null hypothesis and accepting the alternative hypothesis, that is, the residuals are not normally distributed which is not desirable.

Table 16



Many econometricians say that we can accept the model even if it does not follow normal distribution especially when the residual is not serially correlated and does not have any ARCH effect. - Sayed Hossain (2012) and O text online, the forecaster's toolbox (2015)

CONCLUSION

This research uses different volatility models to analyze and estimate the volatility of two emerging stock markets in Africa. It also compared the forecasting performance of the ARCH models using different distributions for the two stock index returns. Botswana Stock Market Index (BSMI) and Nairobi Securities Exchange 20 Share Index(NSE20).

For BSMI daily stock returns, TGARCH (1, 1) model has the smallest AIC and SIC values under the normal distribution and student-t distribution thereby making it the more appropriate model for BSMI than others.

For NSE20 daily stock returns, EGARCH (1, 1) model has the smallest AIC and SIC values and thereby a more appropriate model than others under student-t distribution. No model seem appropriate than the others under the normal distribution.

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